Exponential Entropy Measure Defined On Intuitionistic Fuzzy Set

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Abstract

In the present communication, exponential intuitionistic fuzzy information measures are introduced and characterized axiomatically. To show the effectiveness of the proposed measure, it is compared with the existing measures. Fuzzy discrimination and symmetric discrimination measures are defined and their validity is checked. Important properties of new measures are studied. Their applications in pattern recognition and diagnosis problem of crop disease are discussed.

Keywords

Fuzzy set, Fuzzy information, intuitionistic fuzzy set, Discrimination measure, Pattern cognition.

Introduction

Probability has been traditionally used in modeling uncertainty. Since instigated the idea of fuzzy sets, fuzziness becomes another way to model uncertainty. On the other hand, Classical information theory has been widely used in the literature for representing the uncertainties in the data in the form of classical measure theory. But these measures are valid only for a precise data i.e., where the data related to the system are precisely known. But due to the various constraints in day-to-day life, decision makers may give their judgments under the uncertain and imprecise in nature. Thus, there is always a degree of hesitancy between the preferences of the decision making and hence, the analysis conducted under such circumstances is not ideal and hence does not tell the exact information to the system analyst. To handle this, fuzzy set theory Zadeh [1] has been introduced for handling the uncertainties in the data by defining their membership grades between 0 and 1 corresponding to each element in the universe of discourse. Under these environments, Deluca and Termini [2] proposed a set of axioms for the fuzzy entropy based on the Shannon measure [3]. After their pioneer work, various authors Bhandari and Pal [4], Garg et al. [5], Pal and Pal [6], Verma and Sharma [7], introduced the entropy measure under the fuzzy environment.

But with the growing complexities of the systems day-by-day, it is not possible to give a preference towards the alternative under the different attribute in terms of a single or exact number. Hence, to deal with it, intuitionistic fuzzy set (IFS) theory Attanassov [8], is one of the most permissible theories to handle the uncertainties and impreciseness in the data than the crisp or probability theory. For this, Szmidt and Kacprzyk [9] extended the axioms of Deluca and Termini [2] to IFS environment and defined their corresponding axioms. Later on, corresponding to Deluca and Termini [2] fuzzy entropy measure, Vlachos and Sergiadis [10] extended their measure in the IFS environment. Zhang and Jiang [11] presented a measure of intuitionistic fuzzy entropy based on a generalization of measure of Deluca and Termini [2]. Ye [12] proposed a two entropy measure by extending the work as defined by Parkash et al. [13] gives a simplified form of the Ye [12] entropy. Verma and Sharma [7] proposed exponential order entropy under IFS environment. Thus, it has been concluded that the distance or entropy measures are of key importance in a number of theoretical and applied statistical inference and data processing problems.

Preliminaries: In the following, some needed basic concepts and definitions related to fuzzy sets and intuitionistic fuzzy sets are introduced.

Fuzzy set:

Fuzzy set theory deal with lot of problem of science, engineering and medical for which Zadeh [1] introduced fuzzy set A' defined on a finite universe of discourse $X = (x_1, x_2, \ldots\ldots\ldots\ldots, x_n)$ is given as:

$$A' = \{< x, \mu_{A'}(x) > / x \in X\}$$

where, $\mu_{A'}(x) : X \to [0,1]$ is the membership function of A'. The membership value $\mu_{A'}(x)$ describes the degree of belongingness of $x \in X$ in A'.

Some entropy measures defined on fuzzy set are describe as follow:

De Luca and Termini [2] defined fuzzy set entropy for a fuzzy set A' given by

$$H(A') = \tfrac{1}{n}\sum_{i=1}^{n}[\mu_{A'}(X)\log\mu_{A'}(X) + (1-\mu_{A'}(X))\log(1-\mu_{A'}(X))] \tag{1}$$

Pal and Pal [6] introduced the fuzzy exponential entropy for fuzzy set A' given by

$$e^{H(A')} = \frac{1}{n(\sqrt{e}-1)}\sum\left[(\mu_{A'}(x_i))e^{(1-\mu_{A'}(x_i))} + (1-\mu_{A'}(x_i))e^{(\mu_{A'}(x_i))} - 1\right] \tag{2}$$

Further, Anshu Ohlan defined new parametric generalized exponential measure of information

$$G(A) = \sum_{i=1}^{n}\left[e - \mu_A(x_i)e^{\mu_A(x_i)} - (1-\mu_A(x_i))e^{1-\mu_A(x_i)}\right] \tag{3}$$

Intuitionistic Fuzzy Set:

Atanassov [8] proposed a generalization of fuzzy set characterized as intuitionistic fuzzy set (IFSs). For each element $x \in X$, there exist two characteristic functions for membership and non-membership $\mu_A(x)$ *and* $\nu_A(x)$ respectively an intuitionstic fuzzy set A *in* X is given

$$A = \{< x, \mu_A(x), \nu_A(x) > / x \in X\}$$

where, $\mu_A : X \to [0,1]$, $\nu_A : X \to [0,1]$ with the condition $0 \le \mu_A(x_i) + \nu_A(x_i) \le 1$ for all $x \in X$.

Obviously each fuzzy set may be represented by the following IFS

$$A = \{< x, \mu_A(x), 1-\mu_A(x) > / x \in X\}$$

Li, Lu and Cai proposed a method for transforming Atanassov's intuitionistic fuzzy set into fuzzy sets by distributions hesitations degree equally with membership and non-membership. Let

$A=\{<x,\mu_A(x),\nu_A(x)>/x\in X\}$ be an Atanassov's intuitionistic fuzzy set. Then the fuzzy membership function $\mu_{A'}(x)\ to\ A'$ (A' be the fuzzy set) is given as:

$$\mu_{A'}(x)=\mu_A(x)+\frac{\pi_A(x)}{2}=\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)$$

where $\pi_A(x)=1-\mu_A(x_i)-\nu_A(x_i)$ is the hesitation degree of the element $x\in X\ to\ A.$

Entropy defined on intuitionistic fuzzy set:

Zhang and Jiang [11] presented a measure of intuitionistic fuzzy entropy based on a generalization of measure (1) as

$$E_{ZJ}(A)=-\frac{1}{n}\sum\left[\begin{array}{l}\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\log\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)+\\ \left(1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\right)\log\left(1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\right)\end{array}\right]$$

Verma and Sharma [7] presented a measure of intuitionistic fuzzy entropy based on a generalization of measure in (2) as

$$e^{E(A)}=\frac{1}{n\left(\sqrt{e}-1\right)}\sum\left[\begin{array}{l}\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)e^{1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)}\\ +\left(1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\right)e^{1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)}-1\end{array}\right]$$

Parametric exponential fuzzy entropy is given by Anshu Ohlan on fuzzy set in (3) and we define parametric exponential fuzzy entropy for intuitionistic fuzzy set

$$G(A)=\sum_{i=1}^{n}\left[\begin{array}{l}e-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)e^{\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)}\\ -\left(1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)\right)e^{1-\left(\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}\right)}\end{array}\right] \qquad (4)$$

Theorem: Show that measure $G(A)$ is entropy measure over intuitionistic fuzzy set $A.$

Proof. To prove entropy measure we have to satisfy the four properties $K_1\ to\ K_4$.

K_1 : We have to show that $G(A)$ is minimum iff A is crisp set i.e $\mu_A(x)=0$ and $\nu_A(x)=1$ or $\mu_A(x)=1$ and $\nu_A(x)=0.$

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We take $\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2} = \mathrm{P_A}(x_i)$ and equation (9) will be of the form

$$Q(A)=\sum_{i=1}^{n}\left[e-P_A(x_i)e^{P_A(x_i)}-(1-P_A(x_i))e^{(1-P_A(x_i))}\right]$$

Now $Q(A)=0$ iff $\sum_{i=1}^{n}\left[e-P_A(x_i)e^{P_A(x_i)}-(1-P_A(x_i))e^{(1-P_A(x_i))}\right]=0$

Using the result of Pal and Pal [6] we have

$$\sum_{i=1}^{n}\left[e-P_A(x_i)e^{P_A(x_i)}-(1-P_A(x_i))e^{(1-P_A(x_i))}\right]=0$$

iff $P_A(x_i)=0\, or\, 1\, \forall\, x_i\in X$

this implies $\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}=0$.

Therefore $\mu_A(x_i)-\nu_A(x_i)=1$

Now for all $x_i\in X$ again $\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}$ =0

$$\Rightarrow \nu_A(x_i)-\mu_A(x_i)=1$$

We also know that for all i

$$\mu_A(x_i)+\nu_A(x_i)\le 1$$

Solving the above equations

$\mu_A(x_i)=0$, $\nu_A(x_i)=1$. or $\mu_A(x_i)=1$, $\nu_A(x_i)=0$.

Therefore, $Q(A)=0$ iff either $\mu_A(x_i)=0$ and $\nu_A(x_i)=1$ or $\mu_A(x_i)=1$ and $\nu_A(x_i)=0$ $\forall$ *i.*

K₂ : We have to show that $Q(A)$ is maximum iff $\mu_A(x_i)=\nu_A(x_i)\ \forall\ x_i\in X$.

Suppose that $Q(A)=\sum_{i=1}^{n}k(P_A(x_i))$

where $k(P_A(x_i))=\left[e-P_A(x_i)e^{P_A(x_i)}-(1-P_A(x_i))e^{(1-P_A(x_i))}\right]$

Differentiating $k(P_A(x_i))$ w. r. t. $P_A(x_i)$, we get

$$\frac{\partial k(P_A(x_i))}{\partial P_A(x_i)}=\left[-P_A(x_i)e^{P_A(x_i)}-e^{P_A(x_i)}+(1-P_A(x_i))e^{(1-P_A(x_i))}+e^{(1-P_A(x_i))}\right]$$

$\frac{\partial k(P_A(x_i))}{\partial P_A(x_i)}=0$ hold iff $P_A(x_i)=\frac{1}{2}$

i. e $k\left(\frac{1}{2}\right)=0.$

Hence for maximum value there should be $k\left(\frac{1}{2}\right)<0$.

Now,

$$\frac{\partial^2 k(P_A(x_i))}{\partial P_A^2(x_i)} = -\left[2e^{P_A(x_i)} + P_A(x_i)e^{P_A(x_i)} + 2e^{(1-P_A(x_i))} + (1-P_A(x_i))e^{(1-P_A(x_i))}\right]$$

We know that $\frac{\partial^2 k(P_A)}{\partial P_A^2} \le 0$ at $P_A(x_i)=0.5 \;\forall i$ as shown in figure 1.

This implies $Q(A)$ has maximum value at $P_A(x_i)=0.5$

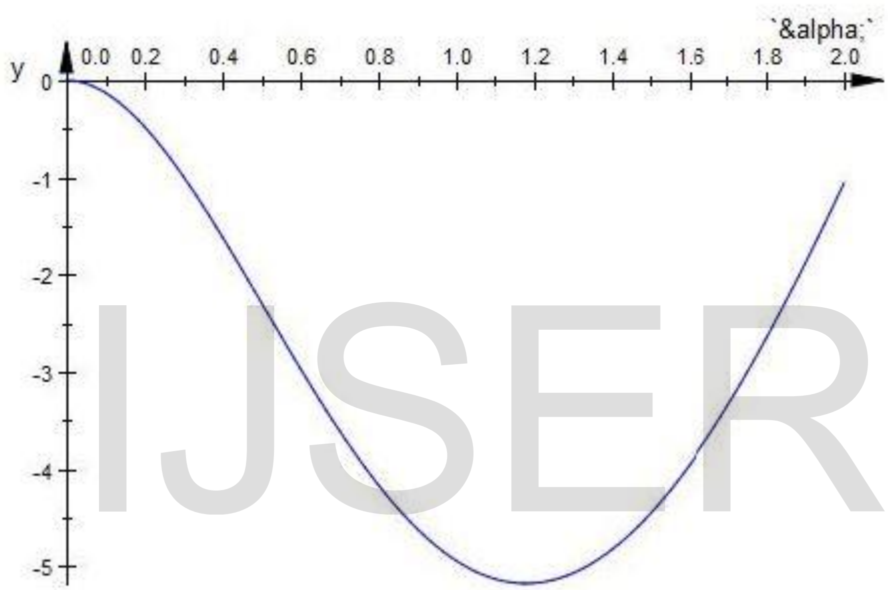

figure 1

***K₃* :** In order prove that $Q(A)$ satisfy the K_3, it is sufficient to prove that function

$$g(p,q)=\left[e-\left(\frac{p+1-q}{2}\right)e^{\left(\frac{p+1-q}{2}\right)}+\left(\frac{q+1-p}{2}\right)e^{\left(\frac{q+1-p}{2}\right)}\right] \text{ where } p,q\in[0,1] \text{ is increasing}$$

w. r. t. p and decreasing for q.

Now,

$$\frac{\partial g}{\partial p}=\left[-\frac{1}{2}e^{\left(\frac{p+1-q}{2}\right)}-\frac{1}{2}\left(\frac{p+1-q}{2}\right)e^{\left(\frac{p+1-q}{2}\right)}-\frac{1}{2}e^{\left(\frac{q+1-p}{2}\right)}-\frac{1}{2}\left(\frac{q+1-p}{2}\right)e^{\left(\frac{q+1-p}{2}\right)}\right]$$

$$\frac{\partial g}{\partial q}=\left[\frac{1}{2}e^{\left(\frac{p+1-q}{2}\right)}+\frac{1}{2}\left(\frac{p+1-q}{2}\right)e^{\left(\frac{p+1-q}{2}\right)}+\frac{1}{2}e^{\left(\frac{q+1-p}{2}\right)}+\frac{1}{2}\left(\frac{q+1-p}{2}\right)e^{\left(\frac{q+1-p}{2}\right)}\right]$$

In order to find the critical point of g, put $\frac{\partial g}{\partial p}=0, \frac{\partial g}{\partial q}=0$

$$\Rightarrow \frac{\partial g}{\partial p}\geq 0 \text{ if } p\leq q \text{ and } \frac{\partial g}{\partial q}\geq 0 \text{ if } p\geq q$$

Therefore, g is increasing function if $p\leq q$ and g is decreasing function if $p\geq q$.

Thus, by containment property, monotonicity of function $g(p,q)$ and we get
$Q(A)\leq Q(B)$ for $A\subseteq B$.

***K*$_4$:** We have $A^C=(<x,\nu_A(x_i),\mu_A(x_i)>/x\in X)$ for all $x_i\in X$ $\nu_A(x_i)=\mu_{A^C}(x_i)$ and $\nu_{A^C}(x_i)=\mu_A(x_i)$. Now clearly by description of complement of intuitionistic fuzzy set and by equation (4), we have, $Q_\alpha(A)=Q_\alpha(A^C)$. Hence $Q(A)$ is a valid entropy measure for intuitionistic fuzzy set.

Example:

Let $\Omega=(x_1,x_2,.......,x_n)$ be a finite universe of discourse and $B=\{\langle x_i,\mu_A(x_i),\nu_A(x_i)\rangle/x_i\in\Omega\}$ be an intuitionistic fuzzy set. We assume that intuitionistic fuzzy set on universal set Ψ which follows as:

$$B^n=\left\{\left\langle x_i,[\mu_A(x_i)]^n,1-[1-\nu_A(x_i)]^n\right\rangle/x_i\in\Omega\right\} \quad (4)$$

We consider an intuitionistic fuzzy set G on Ω which is defined as
$B=\{(x_{1,}\ 0.1,0.8),(x_{2,}\ 0.5,0.4),(x_{3,}\ 0.3,0.5),(x_{4,}\ 0.9,0.0),(x_{5,}\ 1.0,0.0)\}$

Now with the help of operations defined in equation (4) the following intuitionstic fuzzy set are created:

$B^{\frac{1}{2}}, B, B^2, B^3, B^4$

which are defined as follows:

$B^{\frac{1}{2}}$ may be assumed as "More or less LARGE"
B may be assumed as "LARGE"
B^2 may be assumed as "very LARGE"

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B^3 may be assumed as "quite very LARGE"
B^4 may be assumed as "very very LARGE"
and the corresponding set of above notation are given as

$$\mathrm{B}^{\frac{1}{2}} = \begin{Bmatrix} (y_1, 0.3162, 0.5528),\ (y_2, 0.7746, 0.1056), \\ (y_3, 0.5477, 0.2929), (y_4, 0.9487, 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B = \begin{Bmatrix} (y_1, 0.1, 0.8),\ (y_2, 0.5, 0.4), (y_3, 0.3, 0.5), \\ (y_4, 0.9, 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B^2 = \begin{Bmatrix} (y_1, 0.010, 0.9600),\ (y_2, 0.2500, 0.6400), \\ (y_3, 0.0900, 0.7500), (y_4, 0.8100, 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B^3 = \begin{Bmatrix} (y_1, 0.001, 0.9920), (y_2, 0.0125, 0.7840), \\ (y_3, 0.0270, 0.8750), (y_4, 0.7290, 0), (y_5, 1.0, 0) \end{Bmatrix}$$

$$B^4 = \begin{Bmatrix} (y_1, 0.0001, 0.9984),\ (y_2, 0.6250, 0.8704), \\ (y_3, 0.0081, 0.9375), (y_4, 0.6591, 0), (y_5, 1.0, 0) \end{Bmatrix}$$

According to the fuzzy mathematical operation the proposed entropy on different set should be in following order

$$I_R^{\alpha}\left(B^{\frac{1}{2}}\right), I_R^{\alpha}(B),\ I_R^{\alpha}(B^2), I(B^3), I_R^{\alpha}(B^4)$$

Table is constructed for batter comparison between entropies on intuitionistic fuzzy set

	$B^{\frac{1}{2}}$	B	B^2	B^3	B^4
G(B)	2.9256	2.8419	1.9603	1.4487	1.1963

The above table shows that required order of the entropies is followed by the defined entropy.

Conclusion:

Decision making process is incomplete without the use of entropy measure, so in this manuscript, a more generalized intuitionistic fuzzy entropy measure has been presented. Some important properties corresponding to these measures have been studied. The proposed operators show a more stable, practical and optimistic nature to the decision makers during the aggregation process. Further the proposed measure is helpful to solve the multi-criteria decision making problem.

Reference:

[1] L. A. Zadeh, *Fuzzy Sets*. Information and Control, 1965, vol. 8, pp. 338-353.

[2] A. De Luca and S. Termini, A definition of a Non-probabilistic Entropy in Setting of Fuzzy Sets. *Information and Contro*, 1972, vol.20, 301-312.

[3] E. Shannon, A Mathematical Theory of Communication, *The Bell System Technical Journal*, 1948, vol. 27, 379-423.

[4] D. Bhandari and N. R. Pal, Some new information measure for fuzzy set. *Information Science*, 1993, vol.67, 209-228.

[5] H. Garg, Generalized intuitionistic fuzzy multiplicative interactive geometric operators and their application to multiple criteria decision making, *International Journal of Machine Learning and Cybernetics*, 2016, vol.7, 1075-1092.

[6] N. R. Pal and S. K. Pal, Entropy: A new definition and its application of entropy, *IEEE Transaction on system Man and Cybernetics*, 1999, vol. 21, 1260–1270.

[7] R. Verma and B. D. Sharma, Exponential Entropy on Intuitionistic Fuzzy Sets. *Kybernetika*, 2013, vol. 49(1), 114-127.

[8] K. T. Atanassov, Intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, 1986, vol. 20(1), 87-96.

[9] E. Szmidt and J. Kacprzyk, Entropy for Intuitionistic Fuzzy Sets. *Fuzzy Sets and Systems,* 2001, vol.118, 467-477.

[10] I. K. Vlachos and G. D. Sergiadis, Intuitionistic fuzzy information -Applications to pattern recognition. *Pattern Recognition Letters,* 2007, vol. 28, 197-206

[11] Q. S. Zhang and S. Y. Jiang, A note on Information Entropy Measure for Vague Sets. *Inform. Sci.,* 2008, vol. 21, 4184–4191.

[12] J. Ye, Two Effective Measures of Intuitionistic Fuzzy Entropy. *Computing,* 2010, vol. 87, 55-62.

[13] O. Parkash, P. Sharma and R. Mahajan "New measure of weighted fuzzy entropy and their applications for the study of maximum weighted fuzzy entropy principle" *Information Sciences,* vol. 178, pp. 2389-2395, 2008.